Eur Phys J C **33**, s01, s282–s284 (2004)
Digital Object Identifier (DOI) 10.1140/epjcd/s2004-03-1583-y

EPJ C direct
electronic only

Testing the standard model with $B \to (K^*, \rho)\,\gamma$ decays

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Received: 1 December 2004 / Accepted: 18 December 2004 /
Published Online: 8 January 2004 –

Abstract. We review the theory and phenomenology of $B \to (K^*, \rho)\gamma$ decays. The impact of these decay modes on the determination of the CKM angles is investigated. Taking into account most recent experimental data and theoretical inputs, we present an updated analysis of the constraints induced onto the $(\bar{\rho}, \bar{\eta})$ plane.

PACS. 13.20.H2 Decays of bottom mesons – 12.38.Cy Summation of perturbation theory

1 Theoretical setup

The starting point for the analysis of $b \to q\gamma\,(q = d, s)$ transitions is the following effective Hamiltonian

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}}\left[\lambda_q^t \sum_{i=1}^{8} C_i \mathcal{O}_i + \lambda_q^u \sum_{i=1}^{2} C_i (\mathcal{O}_i - \mathcal{O}_i^u)\right], \tag{1}$$

where G_F is the Fermi constant, the explicit CKM factors are $\lambda_q^t = V_{tb}V_{tq}^*$ and $\lambda_q^u = V_{ub}V_{uq}^*$, $\mathcal{O}_i(\mu)$ are dimension-six operators at the scale $\mu \sim O(m_b)$ and $C_i(\mu)$ are the corresponding Wilson coefficients. The latter encode the short distance contributions to the amplitude and do not depend on the particular choice of the external states. New physics can manifest itself only by changing the numerical value of these coefficients or introducing new operators. We refer to [1] for the definition of the operators and a discussion of the Wilson coefficients.

In the context of exclusive decays, we face the difficult task of estimating matrix elements between meson states. A promising approach is the method of QCD-improved factorization that has recently been systemized for non-leptonic decays in the heavy quark limit [2,3]. This method allows for a perturbative calculation of QCD corrections to naive factorization and is the basis for the up-to-date predictions for exclusive rare B decays. However, within this approach, a general, quantitative method to estimate the important $\Lambda_{\rm QCD}/m_b$ corrections to the heavy quark limit is missing. More recently, a more general quantum field theoretical framework for the QCD-improved factorization was proposed - known under the name of Soft–Collinear Effective Theory (SCET) [4,5,6,7,8,9].

Let us now discuss the form of factorization for the decays $B \to V\gamma$ (with $V = K^*,\ \rho$). At leading order, only the operator $\mathcal{O}_7$ contributes and its matrix element between meson states is given by [10,11]

$$F_7^{B\to V} = \xi_\perp^{B\to V} + \phi_B \otimes T \otimes \phi_V + O\left(\frac{\Lambda}{m_b}\right) \tag{2}$$

where $\xi_\perp^{B\to V}$ is the soft contribution to the form factor, $\phi_{B,V}$ are the B and V meson light-cone wave functions and T is a perturbative hard scattering kernel. The advantage of the QCD-improved factorization approach is evident in the computation of the next-to-leading order (in α_s) corrections. In fact, one can show that the matrix elements of the operators $\mathcal{O}_2$ and $\mathcal{O}_8$, which are expected to contribute at this order, are given by the matrix element of $\mathcal{O}_7$ times a computable hard scattering kernel. Moreover, spectator interactions can be computed and are given by convolutions involving the light-cone wave functions of the B and V mesons. It must be mentioned that light-cone wave functions of pseudo-scalar and vector mesons have been deeply studied using light-cone QCD sum rules methods [12,13]. On the other hand, not much is known about the B meson light-cone distribution amplitude, whose first negative moment enters the factorized amplitude at NLO. Since this moment enters the factorized expression for the $B \to \gamma$ form factor as well, it might be possible to extract its value from measurements of decays like $B \to \gamma e\nu$, if it can be shown that power corrections are under control [14].

2 Phenomenology of $B \to (\rho, K^*)\gamma$ decays

In this section we introduce the NLO expressions for the branching ratios and CP asymmetries in $B \to (\rho, K^*)\gamma$ decays. We present the general structure of these observables and refer to [15,16] for a detailed and excplicit description of the various contributions. We discuss only those theoretical inputs whose values have been updated.

In the analysis of exclusive $B \to V\gamma$ decays (with $V = K^*,\ \rho,\ \omega$) we construct the various observables in terms of the CP-averaged quantities - which are much easier to measure than the individual channels - unless otherwise stated. In the NLL approximation, this procedure is equivalent to defining two distinct observables for the charge-conjugate modes and *then* perform the average. The ratios $R(\rho\gamma/K^*\gamma)$ are given by

$$R^{\pm}(\rho\gamma/K^*\gamma) = \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(M_B^2 - M_\rho^2)^3}{(M_B^2 - M_{K^*}^2)^3}\zeta^2(1 + \Delta R^{\pm}) \; ,$$

$$R^{0}(\rho\gamma/K^*\gamma) = \frac{1}{2}\left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(M_B^2 - M_\rho^2)^3}{(M_B^2 - M_{K^*}^2)^3}\zeta^2(1 + \Delta R^{0}) \; ,$$

where $\zeta = \xi_\perp^\rho(0)/\xi_\perp^{K^*}(0)$, and $\xi_\perp^\rho(0)$ and $\xi_\perp^{K^*}(0))$ are the $B \to \rho(K^*)\gamma$ form factors at $q^2 = 0$ [15]. There are several estimates of the quantity ζ in the present literature coming from light-cone QCD sum rules (LCSR) [17], hybrid LCSR [18], improved LCSR [19], quark models [20] and lattice QCD [21]. In the numerical analysis we adopt the value $\zeta = 0.86 \pm 0.10$ in order to accomodate all the above determinations. The quantities $(1 + \Delta R^{\pm,0})$ entail the explicit $O(\alpha_s)$ corrections as well as the power-suppressed annihilation contributions proportional to λ_d^u. The ratio $R(\rho\gamma/K^*\gamma)$ acquires, therefore, a tiny dependence on the CKM angle α. Explicit expressions for these quantities, which are valid in the presence of beyond-the-SM physics, can be found in [15, 22].

Further important observables are the isospin breaking ratio given by

$$\Delta(\rho\gamma) = \frac{\Gamma(B^+ \to \rho^+\gamma) - \Gamma(B^- \to \rho^-\gamma)}{2\,(\Gamma(B^0 \to \rho^0\gamma) + \Gamma(\bar{B}^0 \to \bar{\rho}^0\gamma))} - 1 \qquad (3)$$

and the CP asymmetries in the charged and neutral modes

$$A_{CP}^{\pm}(\rho\gamma) = \frac{\Gamma(B^- \to \rho^-\gamma) - \Gamma(B^+ \to \rho^+\gamma)}{\Gamma(B^- \to \rho^-\gamma) + \Gamma(B^+ \to \rho^+\gamma)} \, , \qquad (4)$$

$$A_{CP}^{0}(\rho\gamma) = \frac{\Gamma(\bar{B}^0 \to \rho^0\gamma) - \Gamma(B^0 \to \rho^0\gamma)}{\Gamma(\bar{B}^0 \to \rho^0\gamma) + \Gamma(B^0 \to \rho^0\gamma)} \, . \qquad (5)$$

On the experimental side, there are only an upper limits on the $B^\pm \to \rho^\pm\gamma$ and $B^0 \to (\rho^0, \omega)\gamma$ modes. They have been combined, using isospin weights for $B \to \rho\gamma$ decays and assuming $\mathcal{B}(B^0 \to \omega\gamma) = \mathcal{B}(B^0 \to \rho^0\gamma)$, to yield the improved 90% C.L. upper limit [23]:

$$R(\rho\gamma/K^*\gamma) \equiv \frac{\mathcal{B}(B \to \rho\gamma)}{\mathcal{B}(B \to K^*\gamma)} < 0.047 \; . \qquad (6)$$

3 Unitarity triangle analysis

Let us present an updated analysis of the constraints in the $(\bar{\rho}, \bar{\eta})$ plane from the unitarity of the CKM matrix, including the measurements of the CP asymmetry $a_{\psi K_s}$ in the decays $B^0/\overline{B^0} \to J/\psi K_s$ (and related modes), and show the impact of the upper limit (6). The SM expressions for ϵ_K (CP-violating parameter in K decays), ΔM_{B_d} (B_d^0–$\bar{B}_d^0$ mass difference), ΔM_{B_s} (B_s^0–$\bar{B}_s^0$ mass difference)

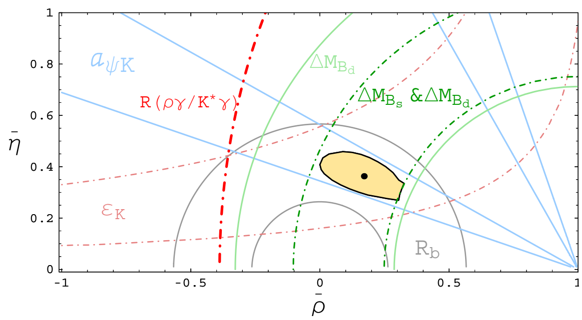

Fig. 1. Unitary triangle fit in the SM and the resulting 95% C.L. contour in the $\bar{\rho}$ - $\bar{\eta}$ plane. The impact of the $R(\rho\gamma/K^*\gamma) < 0.047$ constraint is also shown

and $a_{\psi K_s}$ are fairly standard and can be found, for instance, in [24]. The theoretical parameters and experimental measurements that we use are taken from [22] with the exception of the $B_q - \bar{B}_q$ mixing parameters for which we use the updated values $f_{B_d}\sqrt{B_d} = (210 \pm 24)$MeV and $\xi = 1.19 \pm 0.09$ (here we symmetrize the asymmetric errors induced by the extrapolation of chiral logarithms). The SM fit of the unitarity triangle is presented in Fig. 1.

As the bound from the current upper limit on the ratio $R(\rho\gamma/K^*\gamma)$ is not yet competitive to the ones from either the measurement of ΔM_{B_d} or the current bound on ΔM_{B_s}, we use the allowed $\bar{\rho} - \bar{\eta}$ region to work out the SM predictions for the observables in the radiative B-decays described above. Taking into account these errors and the uncertainties on the theoretical parameters, we find the following SM expectations for the radiative decays [28]:

$$R^{\pm}(\rho\gamma/K^*\gamma) = 0.033 \pm 0.012 \; , \qquad (7)$$

$$R^{0}(\rho\gamma/K^*\gamma) = 0.016 \pm 0.006 \; , \qquad (8)$$

$$\Delta(\rho\gamma) = 0.04^{+0.14}_{-0.07} \; , \qquad (9)$$

$$A_{CP}^{\pm}(\rho\gamma) = -0.10^{+0.02}_{-0.03} \; , \qquad (10)$$

$$A_{CP}^{0}(\rho\gamma) = -0.06 \pm 0.02 \; . \qquad (11)$$

In the CP asymmetries the uncertainties due to formfactors cancel out to a large extent, however, the scale dependence is rather large because the CP asymmetries arise at the $O(\alpha_s)$. The error induced by the imprecise determination of the isospin breaking parameter ζ currently limits the possibility of having a very sharp impact from $R(\rho\gamma/K^*\gamma)$ on the UT analysis.

4 Analysis in supersymmetry

Let us finally discuss the analysis of the exclusive modes in supersymmetric models and entertain two variants of the MSSM called in the literature MFV [26] and Extended-MFV [27] models.

In MFV models, all the flavour changing sources other than the CKM matrix are neglected. In this class of models there are essentially no additional contributions (on top of the SM ones) to $a_{\psi K_S}$ and $\Delta M_{B_s}/\Delta M_{B_d}$, while the

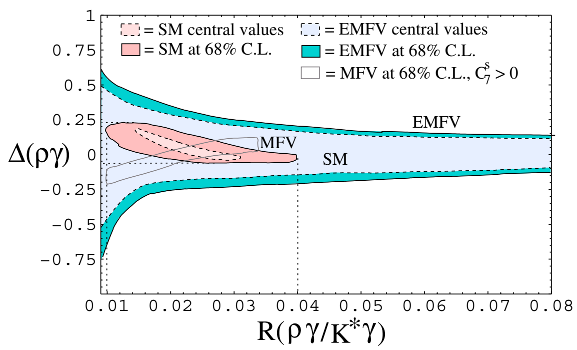

Fig. 2. Correlation between $R(\rho\gamma/K^*\gamma)$ and $\Delta(\rho\gamma)$ in the SM and in MFV and EMFV models. The light-shaded regions are obtained varying $\bar\rho$, $\bar\eta$, the supersymmetric parameters (for the MFV and EMFV models) and using the central values of all the hadronic quantities. The darker regions show the effect of $\pm 1\sigma$ variation of the hadronic parameters

impact on ϵ_K, ΔM_{B_d} and ΔM_{B_s} is described by a single parameter, f, whose value depends on the parameters of the supersymmetric models [24].

EMFV models are based on the assumption that all the superpartners are heavier than 1 TeV with the exception of the lightest stop; no constraints are imposed on the off diagonal structure of the soft breaking terms. It can be shown [27] that under these assumptions there are only two new parameters in addition to the MFV ones, namely: $\delta_{\tilde u_L \tilde t} = M^2_{\tilde u_L \tilde t}/(M_{\tilde t} M_{\tilde q}) \times V_{td}/|V_{td}|$ and $\delta_{\tilde c_L \tilde t} = M^2_{\tilde c_L \tilde t}/(M_{\tilde t} M_{\tilde q}) \times V_{ts}/|V_{ts}|$. Where $\tilde t$ is the lightest stop mass eigenstate and M^2 is the up-squark mass matrix given in a basis obtained from the SCKM one after the diagonalization of the 2×2 stop submatrix. Since we are interested in the phenomenology of $b \to d$ transitions, we will consider here only $\delta_{\tilde u_L \tilde t}$. With the inclusion of this new parameter, the description of the UT-related observables needs one more complex parameter, $g = g_R + i g_I$ [27]. A signature of these models is the presence of a new phase in the $B_d^0 - \bar B_d^0$ mixing amplitude. Using the parametrization $M_{12}^d = r_d^2 e^{2i\theta_d} M_{12}^{\rm SM}$, we get $r_d^2 = |1 + f + g|$ and $\theta_d = 1/2 \arg(1 + f + g)$. This implies new supersymmetric contributions to the CP asymmetry $a_{\psi K_s}$.

The phenomenology of the MFV and EMFV models, analysed by scatter plots over the supersymmetric parameter space, shows the discrimation power of exclusive modes, if one focus on ratios of exclusive observables and their correlation. If one also scan over $\bar\rho$ and $\bar\eta$, and require that each point satisfy the bounds that come from direct searches, from the $B \to X_s\gamma$ branching ratio, and from the UT-related observables, one finally finds the surviving regions presented in Fig. 2. It shows the correlation of the isospin breaking ratio $\Delta(\rho\gamma)$ and the ratio of the branching ratios $R(\rho\gamma/K^*\gamma)$. The light-shaded regions are obtained using the central values of the input parameters while the dark-shaded ones result from the inclusion of their 1σ errors. In the MFV case, there are two distinct regions that correspond to the negative (SM-like) and positive C_7^s case. For $C_7^s < 0$, the allowed regions in MFV almost coincide with the SM ones and we do not draw them. For $C_7^s > 0$, the allowed regions are different and, in general, a change of sign of both the CP-asymmetries (compared to the SM) is expected. We note that the latter scenario needs very large SUSY contributions to C_7^s, arising from the chargino-stop diagrams, and for fixed values of $\tan\beta_S$ it is possible to set an upper limit on the mass of the lightest stop squark.

Acknowledgements. This work is partially supported by the Swiss National Funds. I would like to thank Ahmed Ali and Alexander Parchomenko for interesting discussions.

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